جامعة
بنغازي الحديثة

مجلة جامعة بنغازي الحديثة للعلوم والدراسات الإنسانية

مجلة علمية الكترونية محكمة

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شروط كتابة البحث العلمي في مجلة جامعة بنغازي الحديثة للعلوم والدراسات الإنسانية

1- الملخص باللغة العربية وباللغة الانجليزية (150 كلمة).
2- المقدمة، وتشمل التالي:
❖ نبذة عن موضوع الدراسة (مدخل).
❖ مشكلة الدراسة.
❖ أهمية الدراسة.
❖ أهداف الدراسة.
❖ المنهج العلمي المتبع في الدراسة.
3- الخاتمة. (أهم نتائج البحث - التوصيات).
4- قائمة المصادر والمراجع.
5- عدد صفحات البحث لا تزيد عن (25) صفحة متضمنة الملاحق وقائمة المصادر والمراجع.

القواعد العامة لقبول النشر

1. تقبل المجلة نشر البحوث باللغتين العربية والانجليزية؛ والتي تتوافر فيها الشروط الآتية:

- أن يكون البحث أصيلاً، وتتوافر فيه شروط البحث العلمي المعتمد على الأصول العلمية والمنهجية المتعارف عليها من حيث الإحاطة والاستقصاء والإضافة المعرفية (النتائج) والمنهجية والتوثيق وسلامة اللغة ودقة التعبير.
- ألا يكون البحث قد سبق نشرة أو قُدم للنشر في أي جهة أخرى أو مستل من رسالة أو اطروحة علمية.
- أن يكون البحث مراعياً لقواعد الضبط ودقة الرسوم والأشكال - إن وجدت - ومطبوعاً على ملف وورد، حجم الخط (14) وبخط (Arial 'Body') للغة العربية. وحجم الخط (12) بخط (Times New Roman) للغة الإنجليزية.
- أن تكون الجداول والأشكال مدرجة في أماكنها الصحيحة، وأن تشمل العناوين والبيانات الإيضاحية.
- أن يكون البحث ملتزما بدقة التوثيق حسب دليل جمعية علم النفس الأمريكية (APA) وتثبيت هوامش البحث في نفس الصفحة والمصادر والمراجع في نهاية البحث على النحو الآتي:
- أن تُثبت المراجع بذكر اسم المؤلف، ثم يوضع تاريخ نشرة بين حاصرتين، ويلي ذلك عنوان المصدر، متبوعاً باسم المحقق أو المترجم، ودار النشر، ومكان النشر، ورقم الجزء، ورقم الصفحة.
- عند استخدام الدوريات (المجلات، المؤتمرات العلمية، الندوات) بوصفها مراجع للبحث: يُذكر اسم صاحب المقالة كاملاً، ثم تاريخ النشر بين حاصرتين، ثم عنوان المقالة، ثم ذكر اسم المجلة، ثم رقم المجلد، ثم رقم العدد، ودار النشر، ومكان النشر، ورقم الصفحة.

2. يقدم الباحث ملخص باللغتين العربية والانجليزية في حدود (150 كلمة) بحيث يتضمن مشكلة الدراسة، والهدف الرئيسي للدراسة، ومنهجية الدراسة، ونتائج الدراسة. ووضع الكلمات الرئيسية في نهاية الملخص (خمس كلمات).

3. تحتفظ مجلة جامعة بنغازي الحديثة بحقها في أسلوب إخراج البحث النهائي عند النشر.

إجراءات النشر

ترسل جميع المواد عبر البريد الالكتروني الخاص بالمجلة جامعة بنغازي الحديثة وهو كالتالي:

- ✓ يرسل البحث الكترونياً (Word + Pdf) إلى عنوان المجلة info.jmbush@bmu.edu.ly او نسخة على CD بحيث يظهر في البحث اسم الباحث ولقبة العلمي، ومكان عملة، ومجاله.
- ✓ يرفق مع البحث نموذج تقديم ورقة بحثية للنشر (موجود على موقع المجلة) وكذلك ارفاق موجز للسيرة الذاتية للباحث إلكترونياً.
- ✓ لا يقبل استلام الورقة العلمية الا بشروط وفورمات مجلة جامعة بنغازي الحديثة.
- ✓ في حالة قبول البحث مبدئياً يتم عرضة على مُحكَمين من ذوي الاختصاص في مجال البحث، ويتم اختيارهم بسرية تامة، ولا يُعرض عليهم اسم الباحث أو بياناته، وذلك لإبداء آرائهم حول مدى أصالة البحث، وقيمته العلمية، ومدى التزام الباحث بالمنهجية المتعارف عليها، ويطلب من المحكم تحديد مدى صلاحية البحث للنشر في المجلة من عدمها.
- ✓ يُخطر الباحث بقرار صلاحية بحثه للنشر من عدمها خلال شهرين من تاريخ الاستلام للبحث، وبموعد النشر، ورقم العدد الذي سينشر فيه البحث.
- ✓ في حالة ورود ملاحظات من المحكُمين، تُرسل تلك الملاحظات إلى الباحث لإجراء التعديلات اللازمة بموجبها، على أن تعاد للمجلة خلال مدة أقصاها عشرة أيام.
- ✓ الأبحاث التي لم تتم الموافقة على نشرها لا تعاد إلى الباحثين.
- ✓ الأفكار الواردة فيما ينشر من دراسات وبحوث وعروض تعبر عن أراء أصحابها.
- ✓ لا يجوز نشر إي من المواد المنشورة في المجلة مرة أخرى.
- ✓ يدفع الراغب في نشر بحثه مبلغ قدره (400 د.ل) دينار ليبي إذا كان الباحث من داخل ليبيا، و (200 $) دولار أمريكي إذا كان الباحث من خارج ليبيا. علماً بأن حسابنا القابل للتحويل هو: **(بنغازي – ليبيا - مصرف التجارة والتنمية، الفرع الرئيسي - بنغازي، رقم 001-225540-0011. الاسم (صلاح الأمين عبدالله محمد)**.
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On $\mathcal{G}$A- Sets and $\mathcal{G}$A Continuous Functions in Grill Topological Spaces

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Abstract.

In this paper, We introduce the notion of a regular- $\mathcal{G}$-closed set, $\mathcal{G}$A-set by using regular- $\mathcal{G}$-closed set and study some of its properties, we also investigate the relation between such sets with other grill sets Furthermore, by using $\mathcal{G}$A-set, we obtain new decompositions of continuity. Then we show that a function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is continuous if and only if it is $\mathcal{G}$-α-continuous and $\mathcal{G}$A-continuous.

Keywords: grill topological space, decomposition of continuity, regular closed set, A-set, regular- $\mathcal{G}$-closed set, $\mathcal{G}$A-set.

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الملخص:

في هذا البحث نقدم مفهوم جديد في الفضاء (**Grill Topological Spaces**) من النمط ($\mathcal{G}$A-set , $\mathcal{G}$A-continuous function) واستنتجنا بعض الخواص واثبتنا بعض النظريات المتعلقة به باستخدام مفهوم (regular- $\mathcal{G}$-closed set).

1. Introduction.

The idea of grill on a topological space was first introduced by Choquet [5] in 1947. The concept of grills has shown to be a powerful supporting and useful tool like nets and filters, for getting a deeper insight into further studying some topological notions such as proximity spaces, closure spaces and the theory of compactifications and extension problems of different kinds (see [4], [6], [12] for details). In 2007, Roy and Mukherjee [20] defined and studied a typical topology associated rather naturally to the existing topology and a grill on a given topological space. Quite recently, Hatir and Jafari [9] have defined new classes of sets in a grill topological space and obtained a new decomposition of continuity in terms of grills. In [3], Ravi and Ganesan have defined and studied $\mathcal{G}$-α-open sets and $\mathcal{G}$-α-continuous functions in grill topological spaces. In 2004, A. keskin, T. Noiri and S. yuksel [14] proved that the function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is continuous if and only if it is α-I-continuous and A-I- continuous in ideal topological spaces . We extend this decomposition of continuity in terms of grill.

In this paper, we introduce regular- $\mathcal{G}$-closed set, A-set, $\mathcal{G}$A-continuous functions and investigate the relation between such sets with other grill sets (functions) and obtain a decomposition of continuity. Then we show that a function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is continuous if and only if it is $\mathcal{G}$-α-continuous and $\mathcal{G}$A-continuous.

2. PRELIMINARIES.

Throughout this paper, (X, τ) or X represent a topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and Int(A) denote the closure and the interior of A respectively. The power set of X will be denoted by *P(X)***.** A subset A is said to be regular closed if A = cl(Int(A))**.**

Definition 2.1. [10] A nonempty collection $\mathcal{G}$ of subsets of a topological spaces X is said to be a grill on X if

(1) $\emptyset \notin \mathcal{G}$.

(2) $A \in \mathcal{G}$ and $A \subseteq B \subseteq X$ implies $B \in \mathcal{G}$.

(3) $A, B \subseteq X$ and $A \cup B \in \mathcal{G}$ implies $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

If (X, τ) is a topological space with a grill $\mathcal{G}$ on X. Then, we call it a grill topological space and denotes it by $(X, \tau, \mathcal{G})$.

Definition 2.2 [10] Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X. We define a mapping $\Phi : P(X) \rightarrow P(X)$, as follows:

$\Phi(A) = \Phi_{\mathcal{G}}(A, \tau) = \{x \in X : A \cap U \in \mathcal{G}, \text{ for all } U \in \tau(x)\}$ for each $A \in P(X)$.

The mapping Φ is called the operator associated with the grill $\mathcal{G}$ and the topology τ.

Proposition 2.3 [10] Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X. Then for all $A, B \subseteq X$:

(1) $A \subseteq B$ implies that $\Phi(A) \subseteq \Phi(B)$.

(2) If $\mathcal{G}_1$ and $\mathcal{G}_2$ are two grills on X and $\mathcal{G}_1 \subseteq \mathcal{G}_2$, then $\Phi_{\mathcal{G}_1} \subseteq \Phi_{\mathcal{G}_2}$.

(3) If $A \notin \mathcal{G}$ then $\Phi(A) = \emptyset$.

Proposition 2.4 [10] Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X. Then for all A, B ⊆ X:

(1) $\Phi(A \cup B) = \Phi(A) \cup \Phi(B)$.

(2) $\Phi(A \cap B) \subseteq \Phi(A) \cap \Phi(B)$.

(3) $cl(\Phi(A)) = \Phi(A)$.

(4) $cl(\Phi(A)) \subseteq cl(A)$.

(5) $\Phi(\Phi(A)) \subseteq \Phi(A)$.

Theorem 2.5 [10] Let $\mathcal{G}$ be a grill on a topological space (X, τ). If $U \in \tau$, then $U \cap \Phi(A) = U \cap \Phi(U \cap A)$, for any $A \subseteq X$.

In addition to the conditions of the above theorem, if each nonempty open set is a member of the grill, we have:

Theorem 2.6 [10] If $\mathcal{G}$ is a grill on a space (X, τ) with $\tau \setminus \{\emptyset\} \subseteq \mathcal{G}$, then for all $U \in \tau$, $U \subseteq \Phi(U)$.

Definition 2.7 [10] Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X. We define a map $\Psi: P(X) \rightarrow P(X)$

by $\Psi(A) = A \cup \Phi(A)$, for all $A \in P(X)$.

Theorem 2.8 [10] The above map Ψ satisfies Kuratowski's closure axioms.

Proposition 2.9 [7] Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X. Then for all A, B ⊆ X:

(1) $Int(A) \subseteq Int(\Psi(A))$.

(2) $Int(\Psi(A \cap B)) \subseteq Int(\Psi(A))$.

(3) $Int(\Psi(A)) \subseteq \Psi(A)$.

(4) $A \subseteq B \Rightarrow \Psi(A) \subseteq \Psi(B)$.

Theorem 2.10 [8] Let (X, τ) be a topological space and $\mathcal{G}$ be any grill on X. Then, for any A, B ⊆ X

(1) $\Phi(A) \subseteq \Psi(A) \subseteq cl(A)$.

(2) $A \cup \Psi(Int(A)) \subseteq cl(A)$.

(3) $A \subseteq \Phi(A)$ and $B \subseteq \Phi(B) \Rightarrow \Psi(A \cap B) \subseteq \Psi(A) \cap \Psi(B)$.

Definition 2.11 [10] Corresponding to a grill $\mathcal{G}$ on a topological space (X, τ), there exists a unique topology $\tau_{\mathcal{G}}$ (say) on X given by

$\tau_{\mathcal{G}} = \{U \subseteq X : \Psi(X \backslash U) = X \backslash U\}$, where for any $A \subseteq X$, $\Psi(A) = A \cup \Phi(A) = \tau_{\mathcal{G}}$-clA.

Example 2. 12 Consider the grill topological space $(X, \tau, \mathcal{G})$, where X = {a, b, c}, τ = {∅, {a}, {b, c}, X} and $\mathcal{G}$= {{a}, {c}, {a, c}, {a, b}, {b, c}, X}. Then $\tau_{\mathcal{G}}$ = {∅, {a}, {c}, {a, c}, {b, c}, X}.

Now, we will deal with certain properties concerning the topology $\tau_{\mathcal{G}}$.

Theorem 2.13 [10]

(1) If $\mathcal{G}_1$ and $\mathcal{G}_2$ are two grills on a space X with $\mathcal{G}_1 \subseteq \mathcal{G}_2$, then $\tau_{\mathcal{G}_2} \subseteq \tau_{\mathcal{G}_1}$.

(2) If $\mathcal{G}$ is a grill on a space X and $B \notin \mathcal{G}$, then B is closed in $(X, \tau_{\mathcal{G}})$.

(3) For any subset A of a space X and any grill $\mathcal{G}$ on X, Φ(A) is $\tau_{\mathcal{G}}$-closed.

Proof. (1) Let $U \in \tau_{\mathcal{G}_2}$. Then $\tau_{\mathcal{G}_2}$-cl(X \ U) = Ψ(X \ U)

= (X \ U) $\cup \Phi_{\mathcal{G}_2}$(X \ U) =(X \ U). Thus $\Phi_{\mathcal{G}_2}$(X \ U)⊆ (X \ U). Implies that $\Phi_{\mathcal{G}_1}$(X \ U)⊆ (X \ U). So (X \ U) = $\tau_{\mathcal{G}_1}$-cl(X \ U) and hence $U \in \tau_{\mathcal{G}_1}$.

(2) It is obvious that, for $B \notin \mathcal{G}$, Φ(B)= ∅. Then $\tau_{\mathcal{G}}$-cl(B) = Ψ(B) = B∪ Φ(B) = B. Hence B is $\tau_{\mathcal{G}}$-closed.

(3) We have, Ψ(Φ(A)) = Φ(A) ∪ Φ(Φ(A)) = Φ(A). Thus Φ(A) is $\tau_{\mathcal{G}}$-closed.

Theorem 2.14 [10] Let $(X, \tau, \mathcal{G})$ be a grill topological space. Then

$B(\mathcal{G}, \tau) = \{V \backslash A : V \in \tau, A \notin \mathcal{G}\}$ is an open base for $\tau_{\mathcal{G}}$.

Theorem 2.15 [10] Let $\mathcal{G}$ be a grill on a space (X, τ) and A be any subset of X such that A ⊆ Φ(A). Then

$$\text{clA} = \tau_{\mathcal{G}}\text{-clA} = \text{cl}(\Phi(A)) = \Phi(A).$$

Theorem 2.16 [1] Let $\mathcal{G}$ be a grill on a topological space (X, τ). If U ∈ τ, then U ∩ Ψ(A) ⊆ Ψ(U ∩ A), for any A ⊆ X.

Definition 2.17 A subset A of a topological space X is said to be:

(1) $\alpha -$ open [13] if $A \subseteq \text{Int}\big(\text{cl}(\text{Int}(A))\big)$.

(2) A-set [11] if A = U ∩ V, where U ∈τ and V is regular closed

Definition 2.18 A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is said to be:

(1) $\mathcal{G}$- α-open [1] if $A \subseteq \text{Int}\big(\Psi(\text{Int}(A))\big)$.

(2) $\mathcal{G}$-preopen [9] if A ⊆ Int(Ψ(A)).

(3) Φ-open [9] if A ⊆ Int(Φ(A)).

(4) $\mathcal{G}$-semi-open [1] if A ⊆ Ψ(Int(A)).

Theorem 2.19 [1] Let A be a subset of a grill topological space (X, τ, $\mathcal{G}$).

A subset A of X is $\mathcal{G}$- α-open if and only if it is $\mathcal{G}$ -semi-open and $\mathcal{G}$-preopen.

Corollary 2.20 [1] Let (X, τ, $\mathcal{G}$) be a grill topological space and A a subset of X. If $\mathcal{G}$ = {X}. Then the following hold:

(1) A is $\mathcal{G}$ - α -open if and only if A is open.

(2) A is $\mathcal{G}$ -preopen if and only if A is open.

(3) A is $\mathcal{G}$ -semi-open if and only if A is open.

3. regular- $\mathcal{G}$-closed sets.

Definition 3.1 A subset A of a grill topological space (X, τ, $\mathcal{G}$) is said to be regular- $\mathcal{G}$-closed if A = Φ(Int(A)).

The family of all regular- $\mathcal{G}$-closed sets in a grill topological space (X, τ, $\mathcal{G}$) is denoted by $R_{\mathcal{G}}$C(X).

Example 3.2 Let X = {a, b, c, d}, τ = { Ø, {a, c}, {d}, {a, c, d}, X} and $\mathcal{G}$={{a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X} on X. Let A ={a, b, c}. Then A is a regular- $\mathcal{G}$-closed. Since Φ(Int(A)) = Φ({a, c}) = {a, b, c} = A.

Definition 3.3 [2] A subset A of a grill topological space (X, τ, $\mathcal{G}$) is said to be $\mathcal{G}$-perfect if A = Φ(A).

In Example 3.2, if we let A = {b}, then A is a $\mathcal{G}$-perfect.

Defnition 3.4 [2] A subset A of a grill topological space (X, τ, $\mathcal{G}$)

is called a $\tau_{\mathcal{G}}$-dense-in-itself if A ⊆ Φ(A).

Proposition 3.5 A subset A of a grill topological space (X, τ, $\mathcal{G}$) is regular- $\mathcal{G}$-closed, then A= Ψ(Int(A)).

Proof. Let A be regular- $\mathcal{G}$-closed. Then we have Ψ(Int(A)) = Φ(Int(A)) ∪ Int(A) = A∪ Int(A) = A.

Proposition 3.6 For a subset of a grill topological space (X, τ, $\mathcal{G}$), the following properties are hold:

(1) Every regular- $\mathcal{G}$-closed is a $\mathcal{G}$-perfect.

(2) Every regular- $\mathcal{G}$-closed is $\mathcal{G}$ -semi-open.

Proof (1) Let A be a regular-$\mathcal{G}$-closed. Then we have Φ(Int(A))=A. Since Φ(Int(A)) ⊆ Φ(A), then we have A=Φ(Int(A)) ⊆ Φ(A). On the other hand Φ(A) = Φ(Φ(Int(A))) ⊆ Φ(Int(A)) = A. Therefore, A= Φ(A). This shows that A is -perfect.

(2) Let A be a regular-$\mathcal{G}$-closed. Then we have Φ(Int(A))=A. Since Int(A) ⊆ A we have A= Φ(Int(A)) ∪ Int(A) = Ψ(Int(A)). Therefore, A is $\mathcal{G}$ -semi-open.

Remark 3.7 The converse of Proposition 3.6 need not be true as shown by the following examples.

Example 3.8 (1) Let X = {a, b, c, d}, τ = { Ø, {a, c}, {d}, {a, c, d}, X} and $\mathcal{G}$={{a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X} on X. Let A = {a, c}. Then A is a $\mathcal{G}$-semi-open set, which is not regular- $\mathcal{G}$-closed. Since Ψ(Int(A)) = Φ(Int(A)) ∪ Int(A) = {a, b, c}, so that **A** ⊂ Ψ(Int(A))**.** On the other **hand,** Φ(Int(A)) = Φ({a, c}) = {a, b, c}≠ A and hence A is not.

(2) Let X = {a, b, c}, τ = { Ø, {a}, {a, b}, X} and $\mathcal{G}$ ={{c}, {a, c}, {b, c}, X} on X. Let A = {c}, since Φ(A) ={c}=A. Then A is a $\mathcal{G}$-perfect, which is not regular- $\mathcal{G}$-closed. Because Φ(Int(A)) = Φ(Ø) = Ø$\neq A$.

Proposition 3.9 For any subset A of a space (X, τ, $\mathcal{G}$) with τ \{∅}⊆ $\mathcal{G}$, the following are equivalent:

(1) regular- $\mathcal{G}$-closed.

(2) A is $\mathcal{G}$-semi-open and $\mathcal{G}$-perfect.

(3) A is $\mathcal{G}$-semi-open and $\tau_{\mathcal{G}}$-closed.

Proof. (1) ⇒ (2): Obvious by Proposition 3.6

(2) ⇒ (3): Let A be $\mathcal{G}$-semi-open and $\mathcal{G}$-perfect. Since A is $\mathcal{G}$-perfect, we have Ψ(A) = Φ(A) ∪ A= A. Then A is $\tau_{\mathcal{G}}$-closed.

(3) ⇒ (1): Since A is $\mathcal{G}$-semi-open, we have A⊆ Ψ(Int(A)). By using Theorem 2.6, for Int(A) ∈ τ, we have Int(A)⊆ Φ(Int(A)). Now A ⊆ Ψ(Int(A)) = Φ(Int(A)) ∪ Int(A) ⊆ Φ(Int(A)). Therefore, A⊆ Φ(Int(A)). On the other hand, since A is $\tau_{\mathcal{G}}$-closed, we have Φ(A)⊆A, also Φ(Int(A)) ⊆ Φ(A) ⊆ A. Thus, Φ(Int(A)) ⊆A. This shows that A = Φ(Int(A)).

Corollary 3.10 Every regular- $\mathcal{G}$-closed is a $\tau_{\mathcal{G}}$-dense-in-itself .

Proof. The proof is obvious from Proposition 3.9.

Proposition 3.11 In a grill topological space (X, τ, $\mathcal{G}$), every regular- $\mathcal{G}$-closed set is a regular closed .

Proof. Let A be regular- $\mathcal{G}$-closed set. Then we have Φ(Int (A)) = A. Thus, we obtain that cl (A) = cl (Φ(Int (A))) = Φ(Int (A)) = A , by Theorem 2.4. Additionally, by Theorem 2.10 we have Φ(Int (A)) ⊆ cl (Int (A)), and hence A = Φ(Int (A)) ⊆ cl (Int (A))⊆ cl (A)=A. Then we have A = cl (Int (A)), and hence A is regular closed.

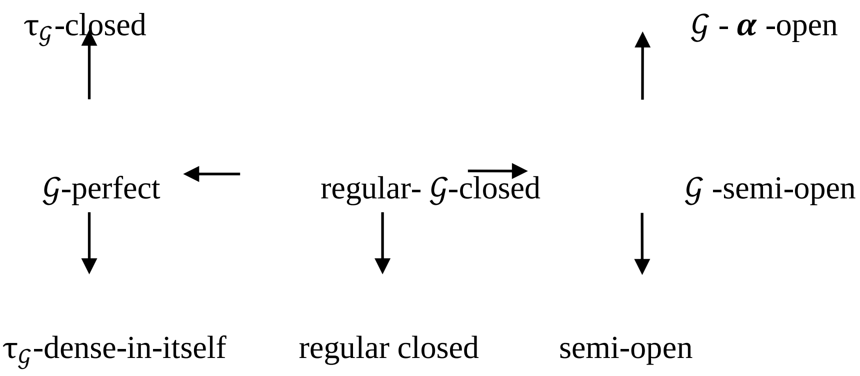
Remark 3.12 The converse of Proposition 3.11 need not be true as shown by the following examples.

Example 3.13 Let X = {a, b, c, d}, τ = { Ø, {a, c}, {d}, {a, c, d}, X} and $\mathcal{G}$={{a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d},{b, c, d}, X} on X. Let A = {b, d}. Then A is a regular closed set, which is not regular- $\mathcal{G}$-closed. Since cl (Int (A)) = cl ({d}) = {b, d}= A , so that A is a regular closed .On the other hand, since Φ(Int(A)) = Φ({d}) = ∅ ≠{ b, d}= A and hen A is not regular- $\mathcal{G}$-closed.

Proposition 3.14 Let $\mathcal{G}$ be a grill on a space (X, τ) with τ \ {∅} ⊆ $\mathcal{G}$**.** Then a subset A of X is a regular- $\mathcal{G}$-closed set *if and only if* A is regular closed.

Proof. By **Proposition 3.11,** every regular- $\mathcal{G}$-closed set is a regular closed. Additionally, by **Theorem** 2,4 and **Theorem** 2.15 we have Φ(Int (A)) = cl(Int (A)). Thus, regular- $\mathcal{G}$-closed and a regular closed are equivalent.

Proposition 3.15 Let (X, τ, $\mathcal{G}$) be a grill topological space. For a subset of X the following implications hold:



4. $\mathcal{G}$A-set.

Definition 4.1 A subset A of a grill topological space (X, τ, $\mathcal{G}$) is said to be $\mathcal{G}$A-set if A = U ∩ V, where U ∈τ and V ∈ $R_{\mathcal{G}}$C(X).

The family of all $\mathcal{G}$A-set in a grill topological space (X, τ, $\mathcal{G}$) is denoted by $\mathcal{G}$A (X).

Proposition 4.2 Every regular- $\mathcal{G}$-closed is $\mathcal{G}$A-set.

Proof. Since X ∈ $\tau \cap R_{\mathcal{G}}$C(X) , the proof is obvious .

Remark 4.3 The converse of Proposition 4.2 need not be true as shown by the following examples.

Example 4.4 (1) Let X = {a, b, c, d}, τ = { Ø, {a, c}, {d}, {a, c, d}, X} and $\mathcal{G}$={{a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X} on X. Let A= {a, c}.Then A is not a regular- $\mathcal{G}$-closed set, and let V = {a, b, c}. Then by Example 3.2, V is regular- $\mathcal{G}$-closed and A is open . Therefore, A = A ∩ V is $\mathcal{G}$A-set.

Definition 4.5 [3] A subset A of a grill topological space (X, τ, $\mathcal{G}$) is said to be weakly $\mathcal{G}$-locally closed if A = U ∩ V, where U ∈τ and V is a $\tau_{\mathcal{G}}$-closed set.

Proposition 4.6 Let A be a subset of a grill topological space (X, τ, $\mathcal{G}$). Then the following properties hold:

(1) If A is $\mathcal{G}$A-set, then A is weakly $\mathcal{G}$-locally closed .
(2) If A is $\mathcal{G}$A-set, then A is an A-set.

Proof. obvious by Proposition 3.9, and Proposition 3.11.

Remark 4.7 The converse of Proposition 4.6 need not be true as shown by the following examples.

Examples 4.8

(1) Consider the grill topological space (X, τ, $\mathcal{G}$) of Examples 3.8 (2). If A ={c}. Then A is weakly $\mathcal{G}$-locally closed, which is not a $\mathcal{G}$A-set, we have A=X∩A such that Ψ(A) =A. Therefore A is weakly $\mathcal{G}$-locally closed. On the other hand, we have A is not regular- $\mathcal{G}$-closed.

(2) Let A = {b, d}. Then by Example 3.13, A is a regular closed set, which is not regular- $\mathcal{G}$-closed. Therefore A is an A-set which is not A-set**.**

Proposition 4.9 Let (X, τ, $\mathcal{G}$) be a grill topological space and A a subset of X. If $\mathcal{G}$ = {X}. Then the following are equivalent:

(1) A is an open set,

(2) A is a $\mathcal{G}$- α-open and a A-set,

(3) A is a $\mathcal{G}$-preopen and a $\mathcal{G}$A-set.

Proof. (1) ⇒ (2): Let A is an open. Hence A is $\mathcal{G}$- α-open by Proposition 2.19, On the other hand, we have A= A∩X**, where** A∈ τ**,** and X is regular- $\mathcal{G}$-closed. Hence A is $\mathcal{G}$A-set.

(2)⇒ (3): This is obvious since every $\mathcal{G}$- α-open set is $\mathcal{G}$-preopen by Corollary 2.20 .

(3)⇒(1): Let A is a $\mathcal{G}$ -preopen and a $\mathcal{G}$A-set. Then we have A⊆ Int(Ψ(A)) and A= U∩V, where U∈ τ and V is regular- $\mathcal{G}$-closed, respectively. Therefore, we have

$$A \subseteq Int(\Psi(A)) = Int(\Psi(U \cap V))$$
$$\subseteq Int(\Psi(U) \cap \Psi(V))$$

$= Int(\Psi(U)) \cap Int(\Psi(V))$.

Since V is regular- $\mathcal{G}$-closed, then V is a $\tau_{\mathcal{G}}$-closed and $\Psi(V)=V$, implies that $A = Int(\Psi(U)) \cap Int((V))$.

Since $A=U\cap V$ and $A\subseteq U$, we have

$$A = U\cap A \subseteq U\cap Int(\Psi(U))\cap Int((V))$$
$$= [U\cap Int(\Psi(U))]\cap Int(V)$$
$$= Int[U\cap \Psi(U)]\cap Int(V)$$
$$= U\cap Int(V)$$
$$= Int(U\cap V)$$
$$= Int(A).$$

This shows that $A\in \tau$.

5. Decomposition of Continuity.

Definition 5.1 A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is said to be $\mathcal{G}$-$\boldsymbol{\alpha}$-continuous [1] (resp. $\mathcal{G}$-semi-continuous [1], $\mathcal{G}$-pre-continuous [9], contra weakly $\mathcal{G}$-LC- continuous [3], A-continuous [11]) if the inverse image of each open set of Y is $\mathcal{G}$- $\boldsymbol{\alpha}$ -open (resp. $\mathcal{G}$ -semi-open, $\mathcal{G}$–preopen, weakly $\mathcal{G}$-locally closed, A-set) in $(X, \tau, \mathcal{G})$, respectively.

Definition 5.2 A function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ is said to be $\mathcal{G}$A- continuous if the inverse image of each open set of Y is $\mathcal{G}$A- set of $(X, \tau, \mathcal{G})$.

Proposition 5.3 For a function $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$, the following properties are hold:

(1) if f is $\mathcal{G}$A-continuous, then f is a contra weakly $\mathcal{G}$-locally closed continuous.

(2) if f is $\mathcal{G}$A-continuous, then f is a A- continuous.

Proof. The proof is obvious by Proposition 4.6.

Remark 5.4 The converse of Proposition 5.3 need not be true as shown by the following examples.

Example 5.5 Let $X = \{a, b, c, d\}$, $\tau = \{ \emptyset, \{a, c\}, \{d\}, \{a, c, d\}, X\}$ and $\mathcal{G} = \{\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ on X. Let $Y= \{a, b\}$ and $\sigma =\{\emptyset, \{a\}, Y\}$.

(1) Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function defined as follows: $f(a) = f(b) = f(d) = b$ and $f(c) = a$. Then f is weakly $\mathcal{G}$-locally closed continuous, which is not $\mathcal{G}$A - continuous by Example 4.8(1).

(2) Let $f : (X, \tau, \mathcal{G}) \rightarrow (Y, \sigma)$ be a function defined as follows: $f(b) = f(d) = a$ and $f(c) = f(a) = b$. Then f is a A- continuous, which is not $\mathcal{G}$A-continuous by Example 4.8(2).

Theorem 5.6 Let (X, τ, $\mathcal{G}$) be a grill topological space and A a subset of X. If $\mathcal{G}$ = {X}. For a function f : (X, τ, $\mathcal{G}$) → (Y, σ), the following properties are equivalent:

(1) f is continuous.

(2) f is a $\mathcal{G}$ - α - continuous and a $\mathcal{G}$A- continuous.

(3) f is a $\mathcal{G}$ -pre continuous and a $\mathcal{G}$A- continuous.

Proof. The proof is obvious by Proposition 4.9.

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